PROOF OF FACTORIZATION WITH SCET

-The factorization of the dijet cross section in electron-positron scattering with jet algorithms

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THE FACTORIZATION THEOREM

$$\sigma = H(Q^2, \mu) J_n(\mu) \otimes J_{\overline{n}}(\mu) \otimes S(\mu)$$

Q. Are they IR finte?
= Is this factorization proper?

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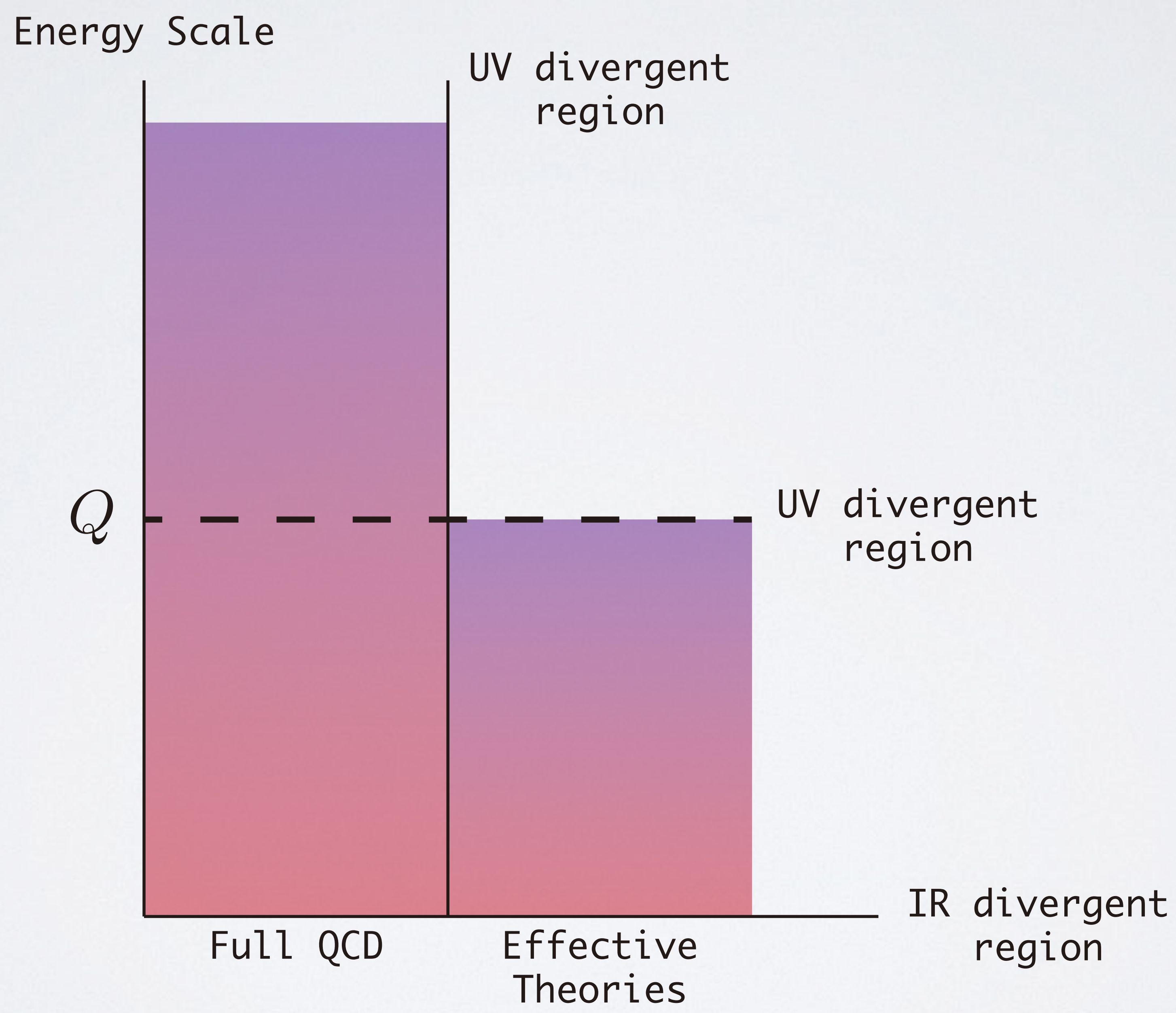
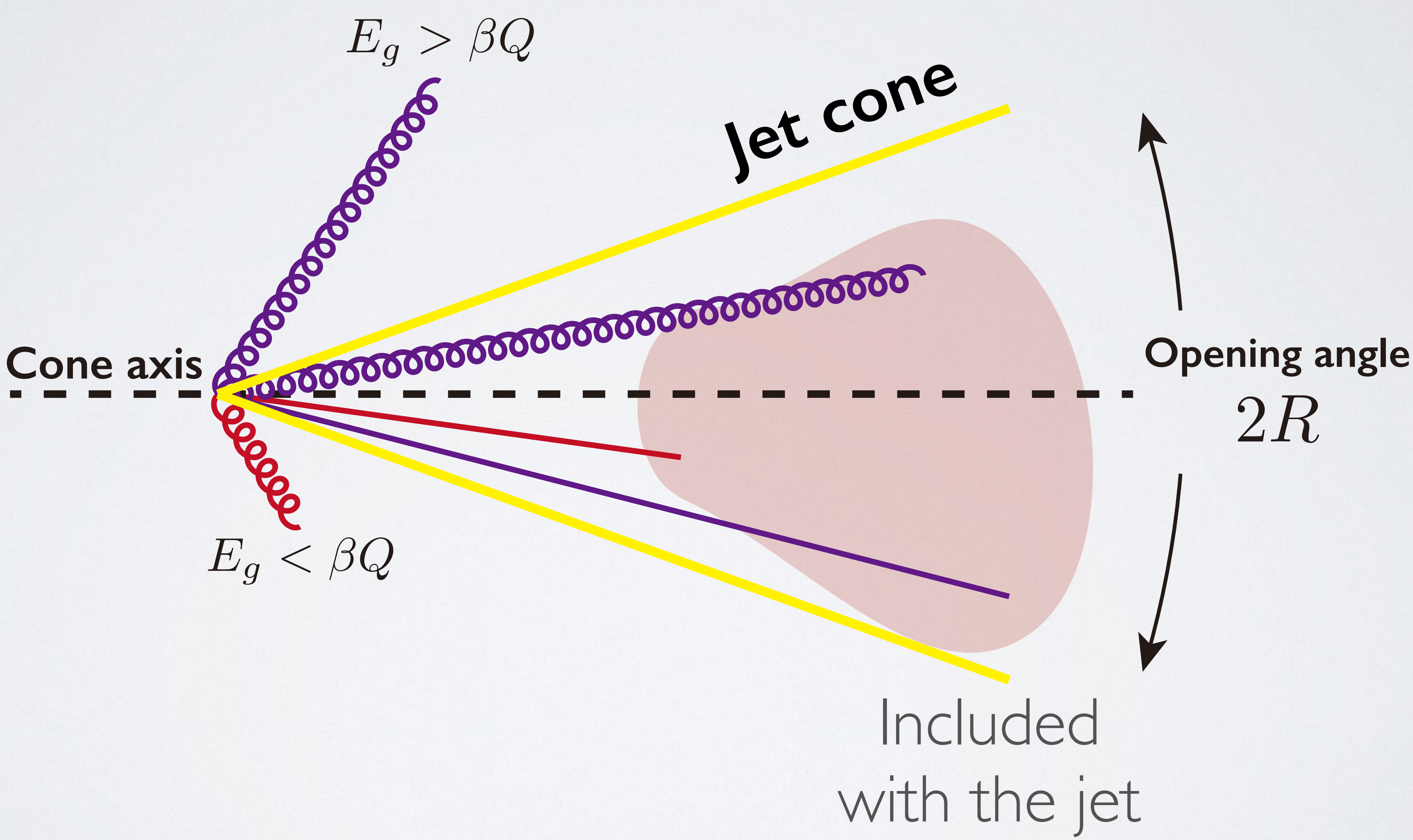
Q. Are they IR finte?
= Is this factorization proper?

A. Check with the soft-collinear effective theory!

LOW ENERGY REGION OF SCET



CONE JET ALGORITHM



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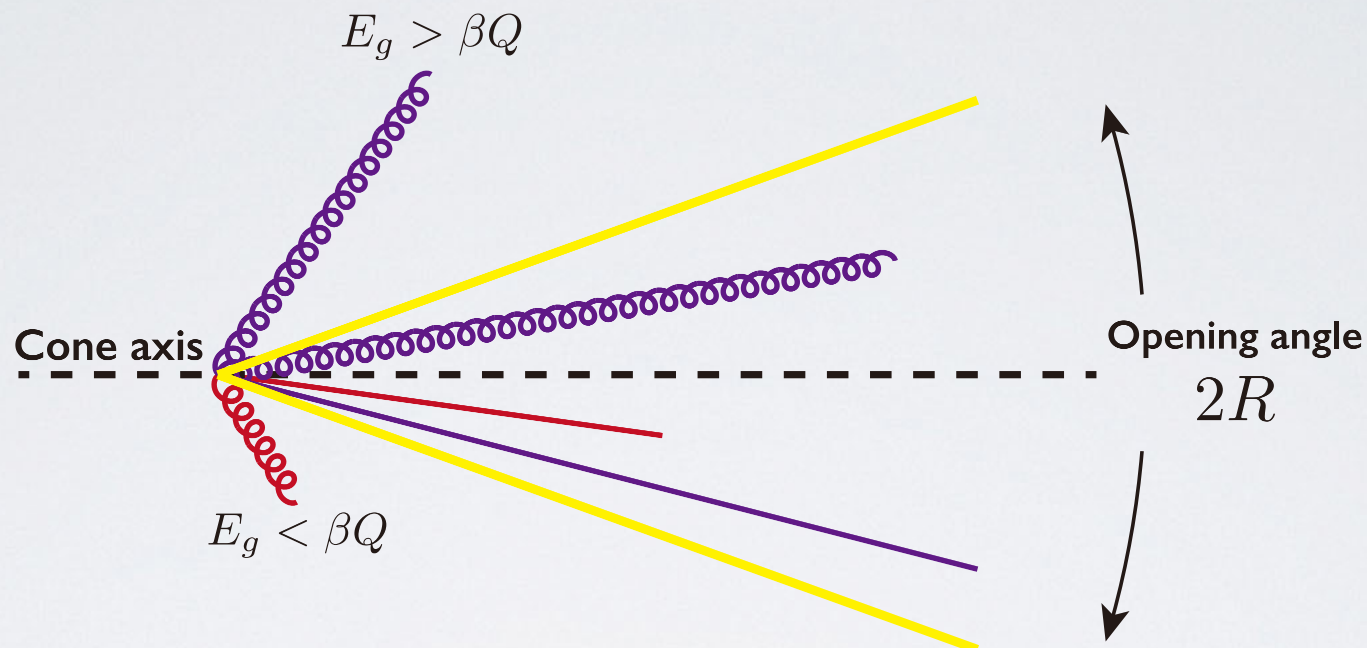
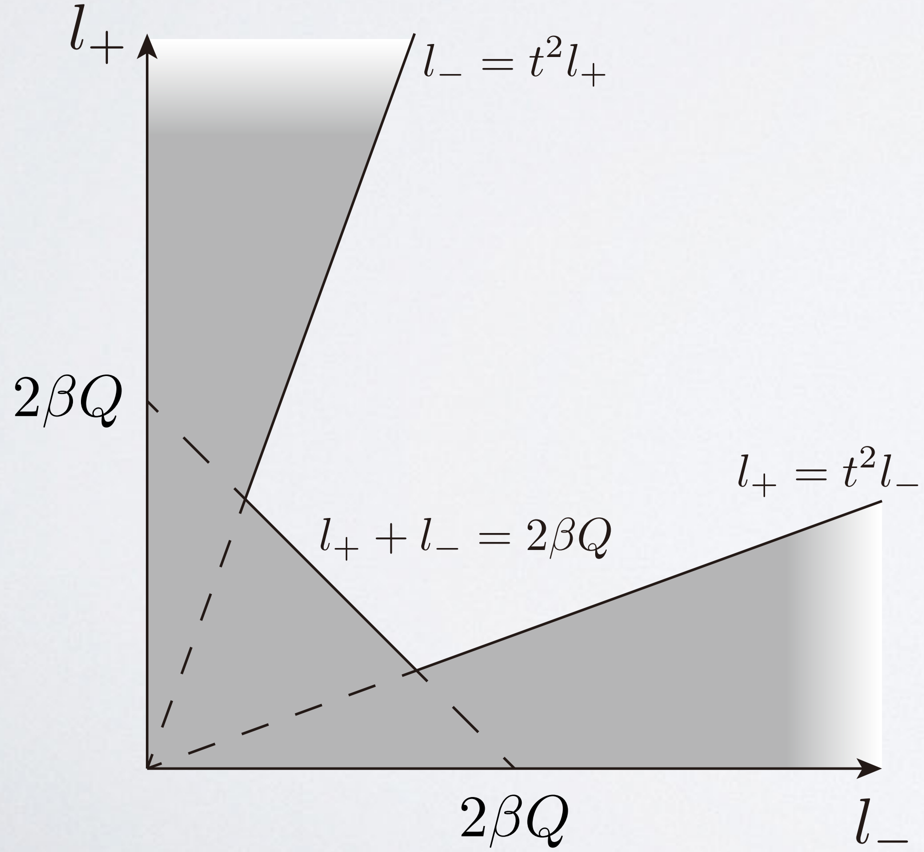
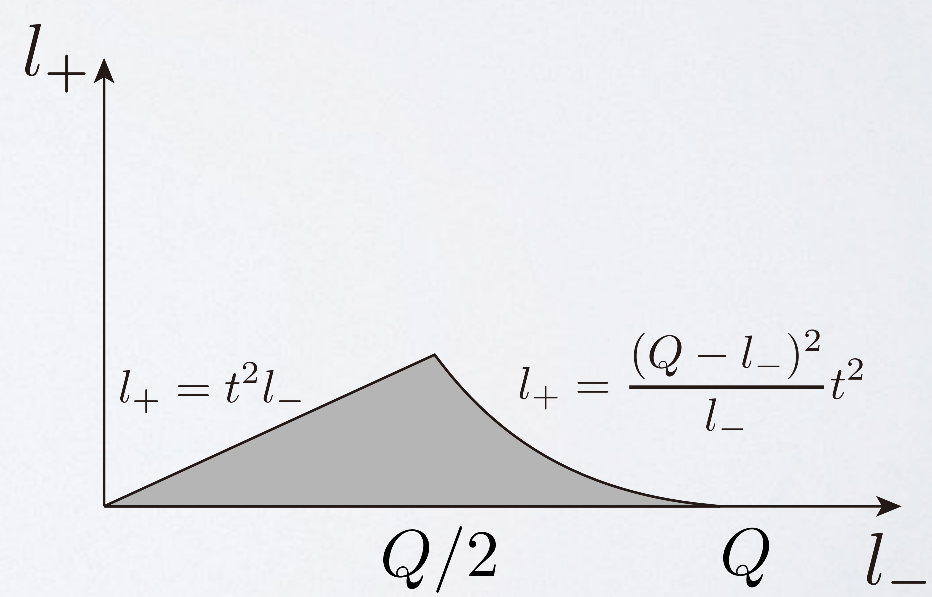
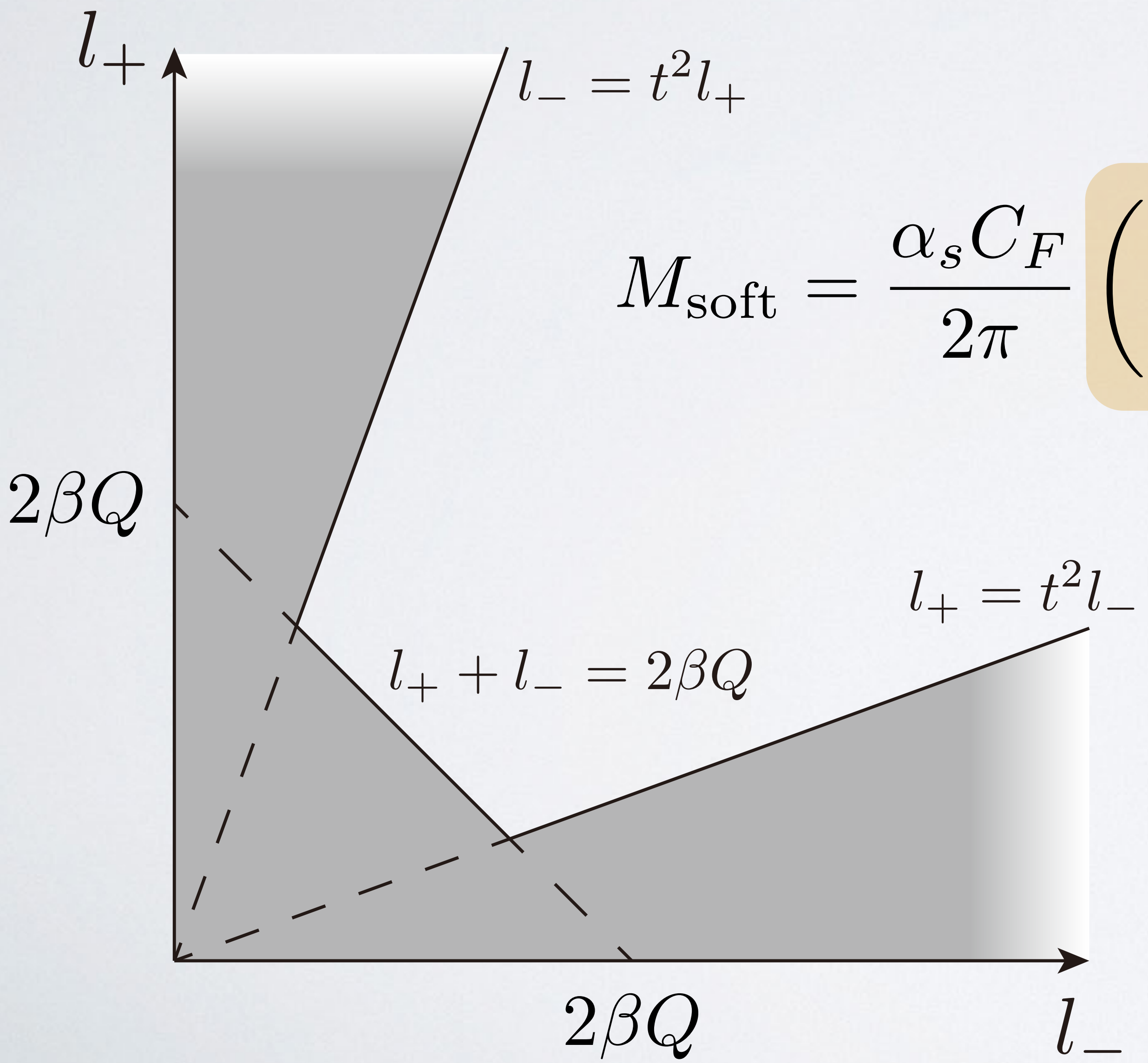






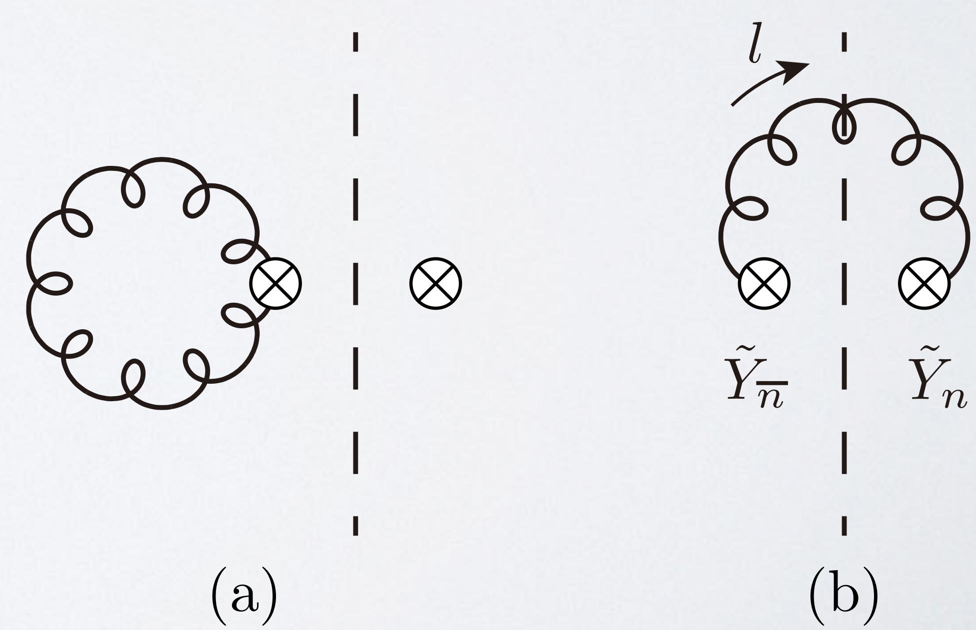
THE SOFT FUNCTION

$$S_\Theta = \sum_{X_s} \frac{1}{N_c} \mathrm{Tr}\langle 0|\tilde{Y}_{\overline{n}}^\dagger \tilde{Y}_n|X_s\rangle \Theta_{\rm soft} \langle X_s|\tilde{Y}_n^\dagger \tilde{Y}_{\overline{n}}|0\rangle.$$



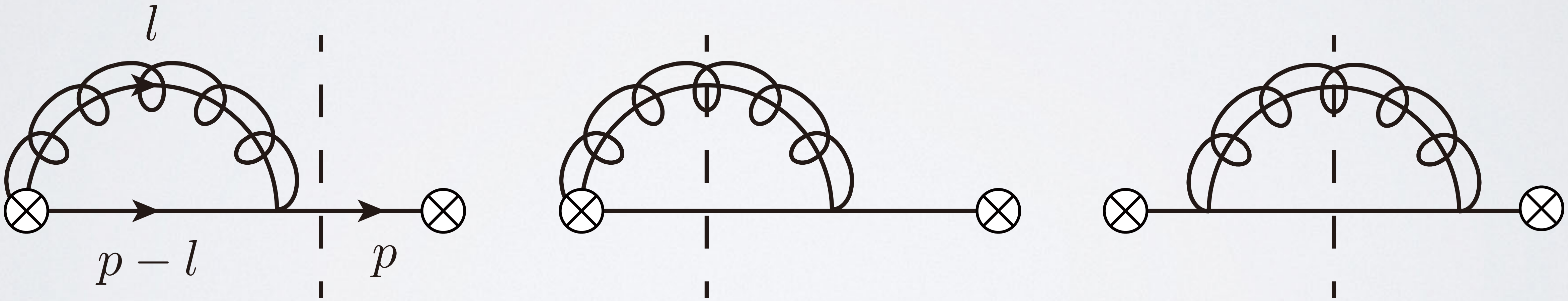
IR finite!

$$M_{\rm soft} = \frac{\alpha_s C_F}{2\pi} \boxed{\left(\frac{4}{\epsilon_{\rm UV}} \ln t + 4 \ln \frac{\mu^2}{4\beta^2 Q^2} \ln t - 4 \ln^2 t - \frac{\pi^2}{3} \right)}$$



THE JET FUNCTION - DEFINITION AND FEYNMAN GRAPHS

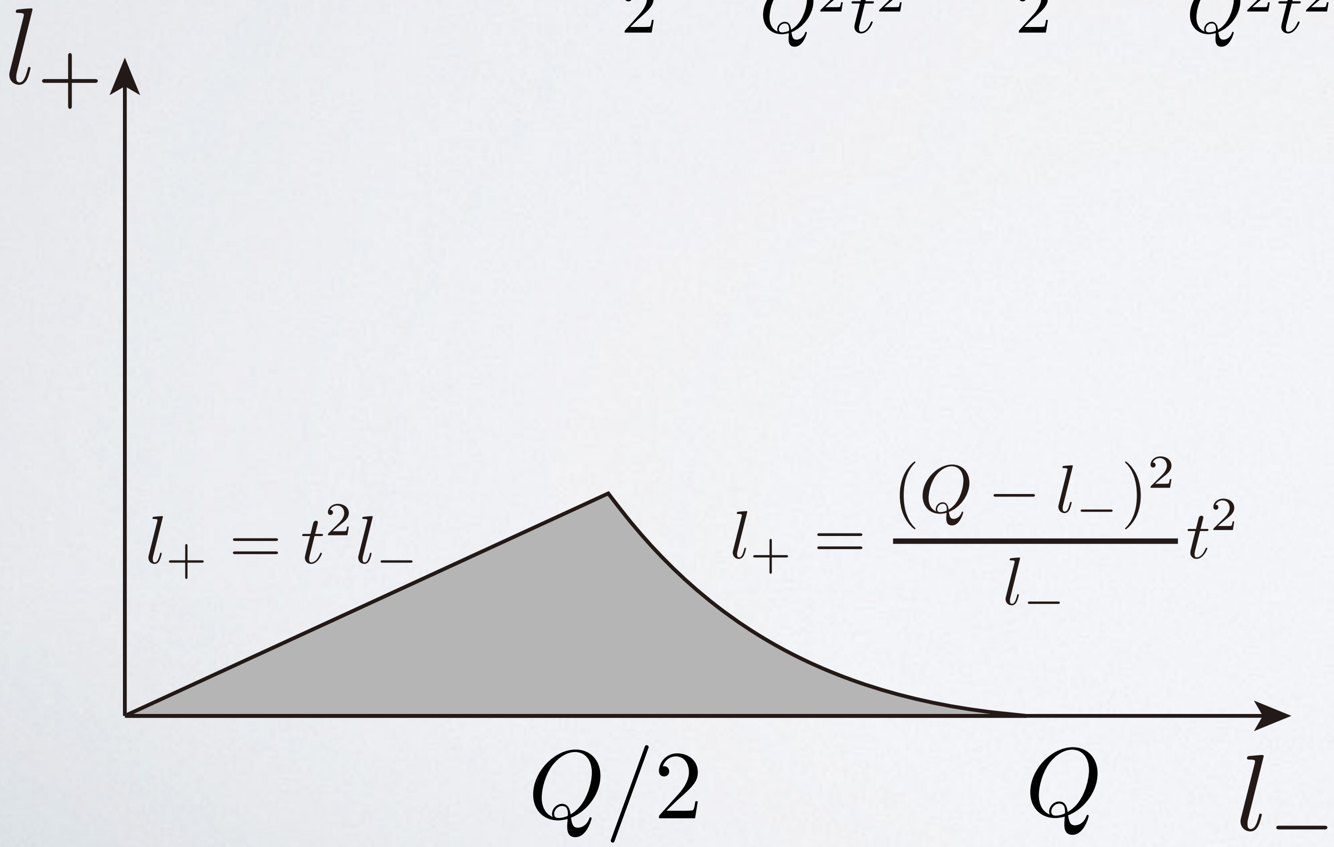
$$\sum_{X_n} \langle 0|\chi_n^{\alpha}|X_n\rangle \Theta_{\text{cone}} \langle X_n|\overline{\chi}_n^{\beta}|0\rangle = \int \frac{d^4 p_{X_n}}{(2\pi)^3} \overline{n}\cdot p_{X_n} \frac{\not{n}}{2} J_n^{\text{cone}}(p_{X_n}^2, \mu)\delta^{\alpha\beta},$$



(a) (b) (c)

THE JET FUNCTION - NAIVE COLLINEAR CONTRIBUTION

$$\tilde{M}_{\mathrm{coll}} = \frac{\alpha_s C_F}{2\pi}\Bigg[-\frac{1}{\epsilon_{\mathrm{IR}}^2}+\frac{2}{\epsilon_{\mathrm{IR}}\epsilon_{\mathrm{UV}}}+\frac{1}{\epsilon_{\mathrm{UV}}}\left(\frac{3}{2}+\ln\frac{\mu^2}{Q^2t^2}\right)+2\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\frac{1}{\epsilon_{\mathrm{IR}}}\right)\ln t$$
$$+\frac{3}{2}\ln\frac{\mu^2}{Q^2t^2}+\frac{1}{2}\ln^2\frac{\mu^2}{Q^2t^2}+\frac{7}{2}+3\ln 2-\frac{5\pi^2}{12}\Bigg]$$

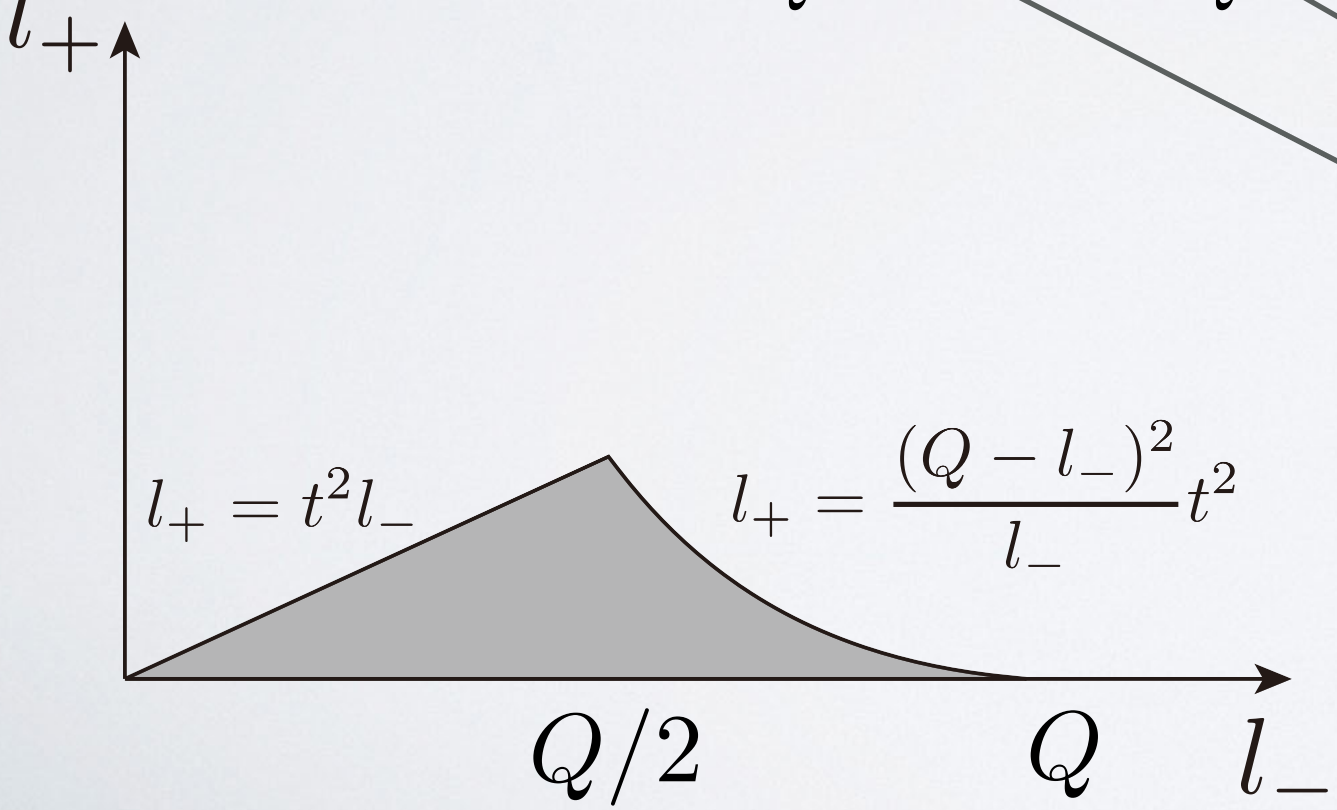


$$l_+ \sim Q\lambda^2,\ l_- \sim Q$$

THE JET FUNCTION
- NAIVE COLLINEAR CONTRIBUTION

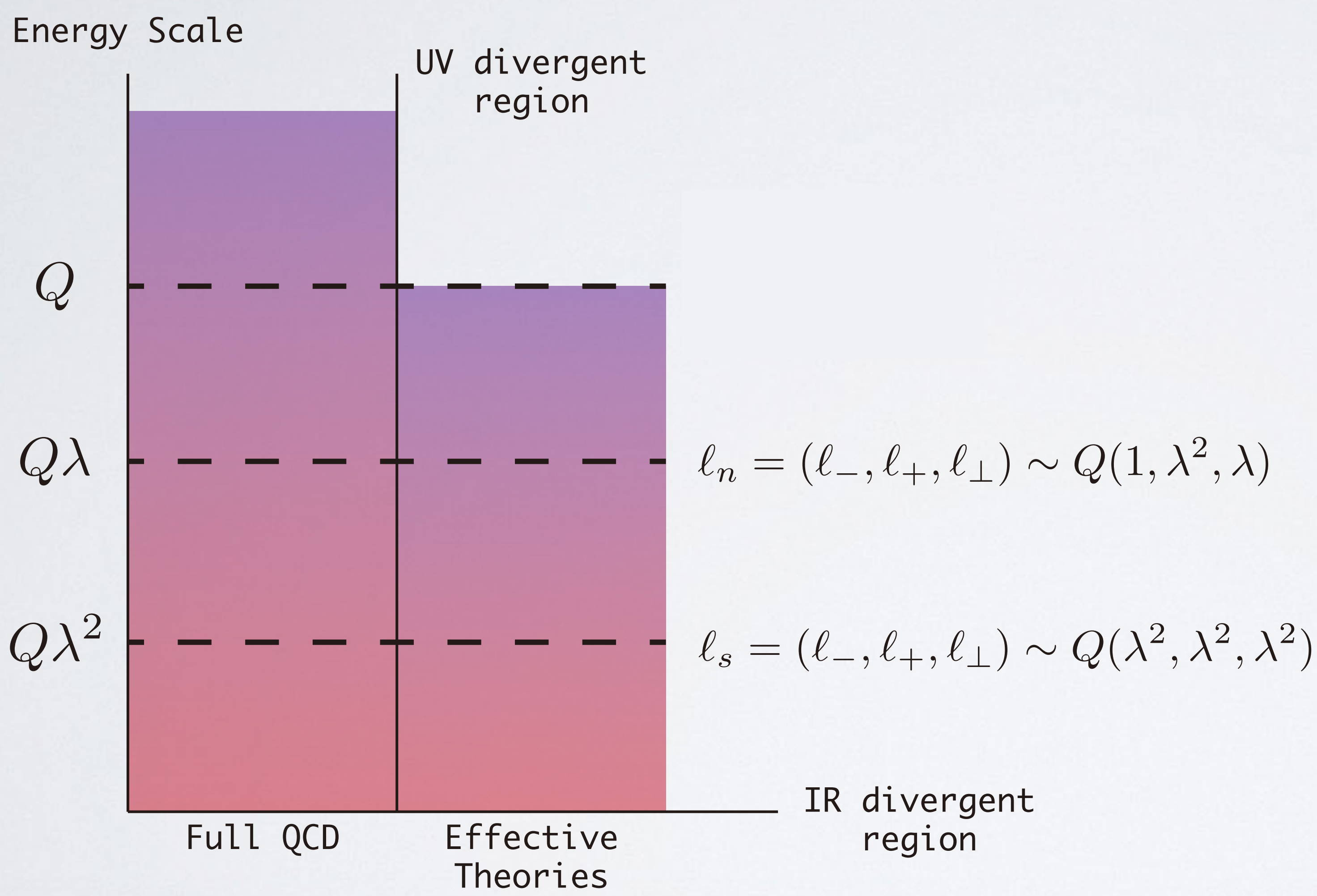
$$\tilde{M}_{\mathrm{coll}} = \frac{\alpha_s C_F}{2\pi}\left[-\frac{1}{\epsilon_{\mathrm{IR}}^2}+\frac{2}{\epsilon_{\mathrm{IR}}\epsilon_{\mathrm{UV}}}+\frac{1}{\epsilon_{\mathrm{UV}}}\left(\frac{3}{2}+\ln\frac{\mu^2}{Q^2t^2}\right)+2\left(\frac{1}{\epsilon_{\mathrm{UV}}}-\frac{1}{\epsilon_{\mathrm{IR}}}\right)\ln t \right.$$
$$\left. +\frac{3}{2}\ln\frac{\mu^2}{Q^2t^2}+\frac{1}{2}\ln^2\frac{\mu^2}{Q^2t^2}+\frac{7}{2}+3\ln 2-\frac{5\pi^2}{12}\right]$$

IR divergent!



$$l_+ \sim Q\lambda^2,\ l_- \sim Q$$

MODE COUNTING



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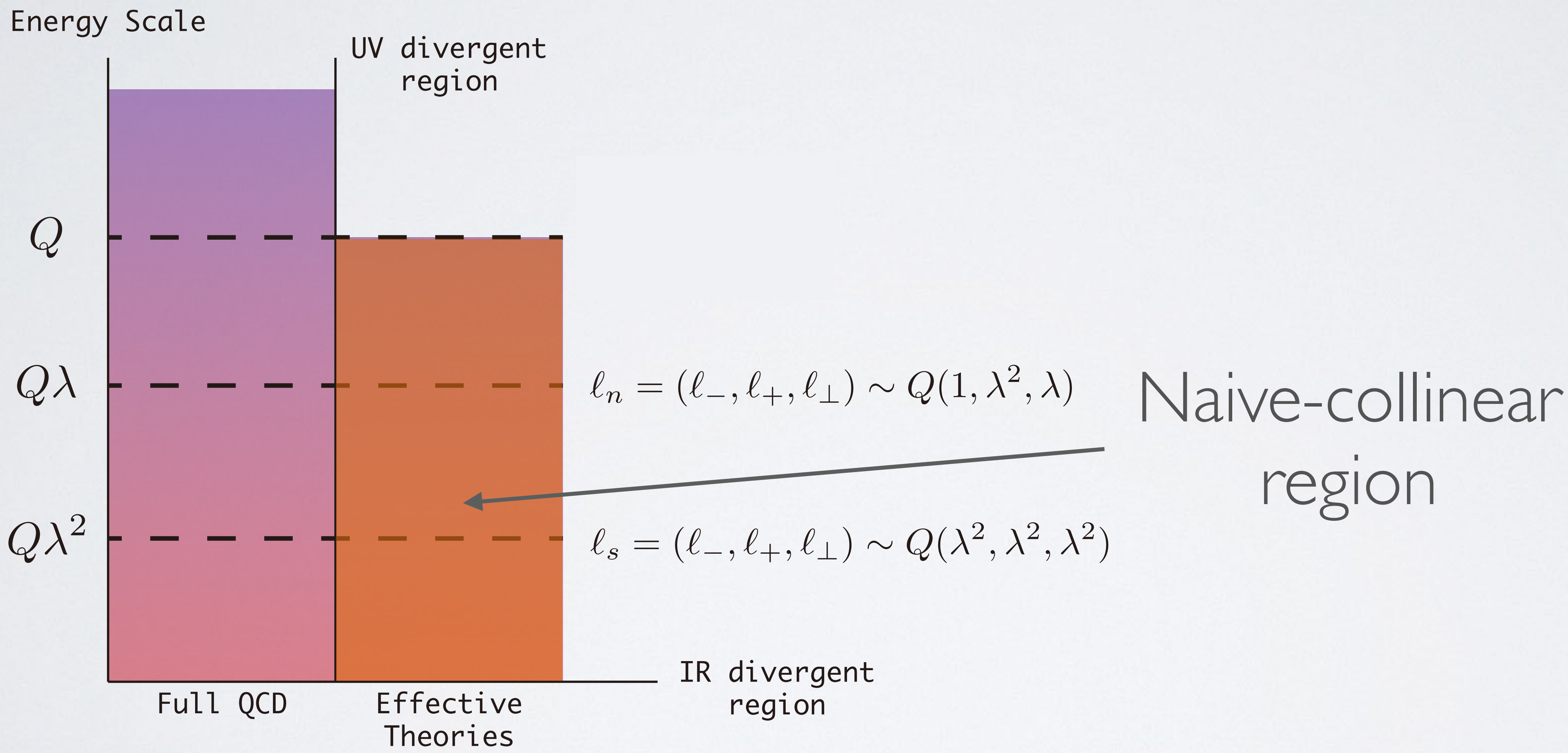
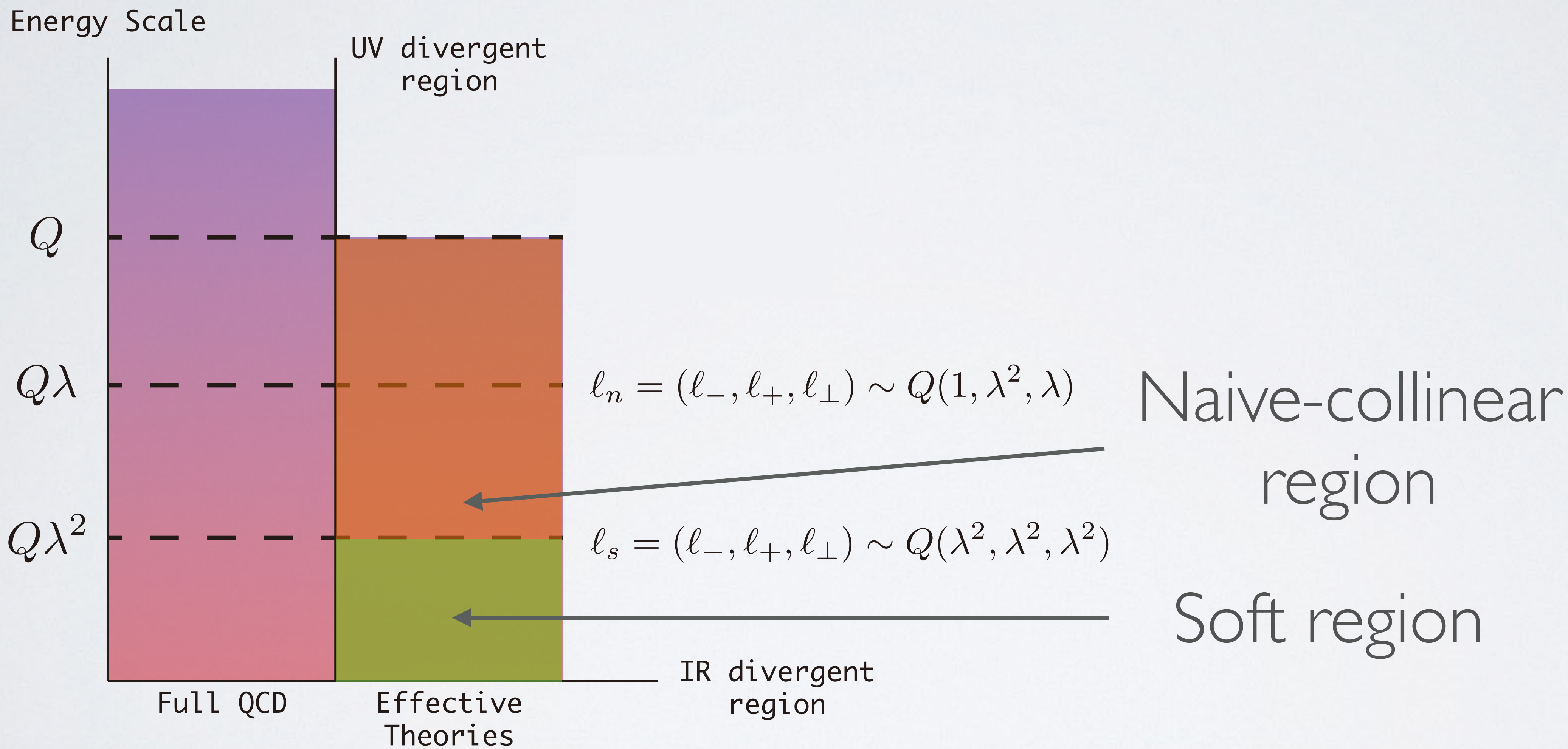
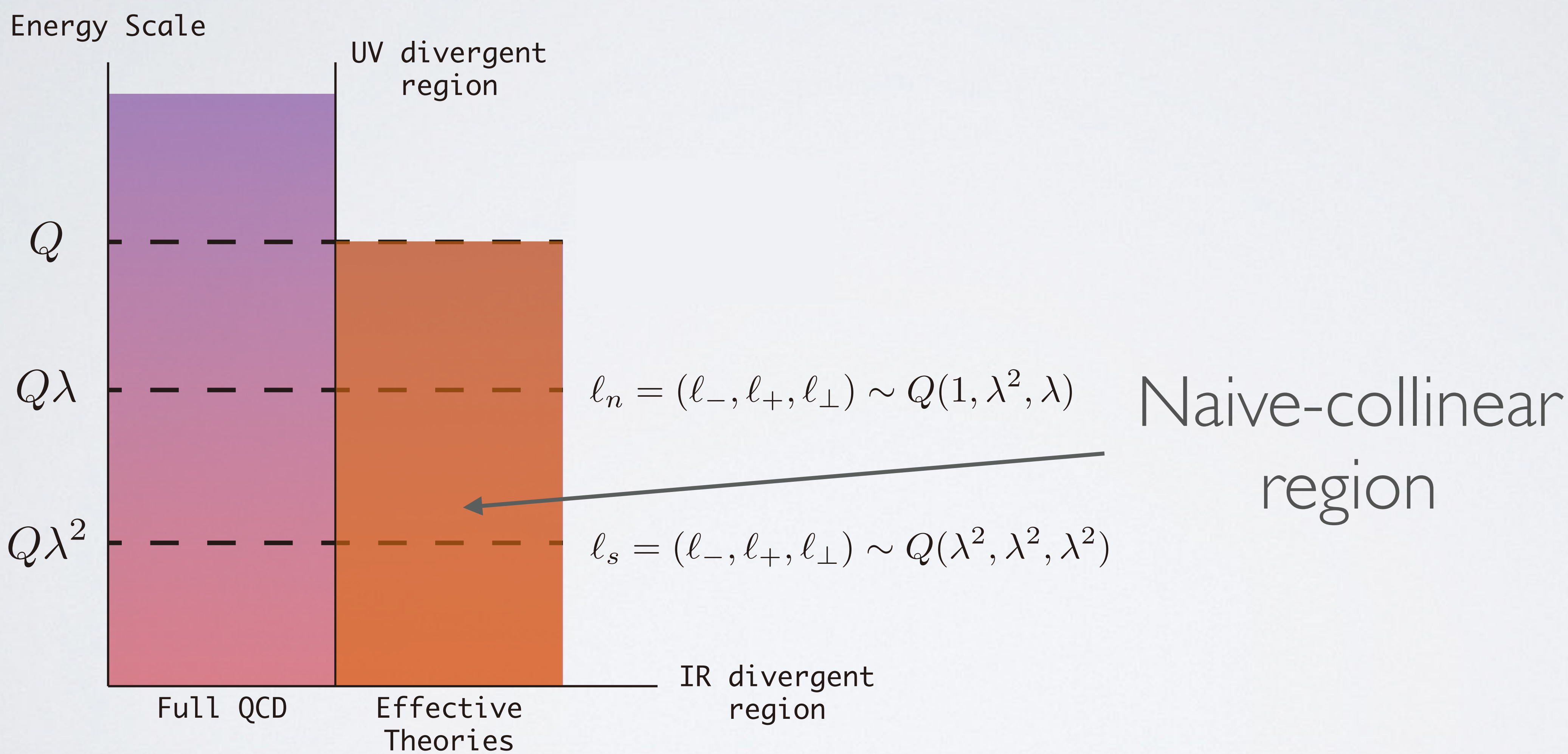


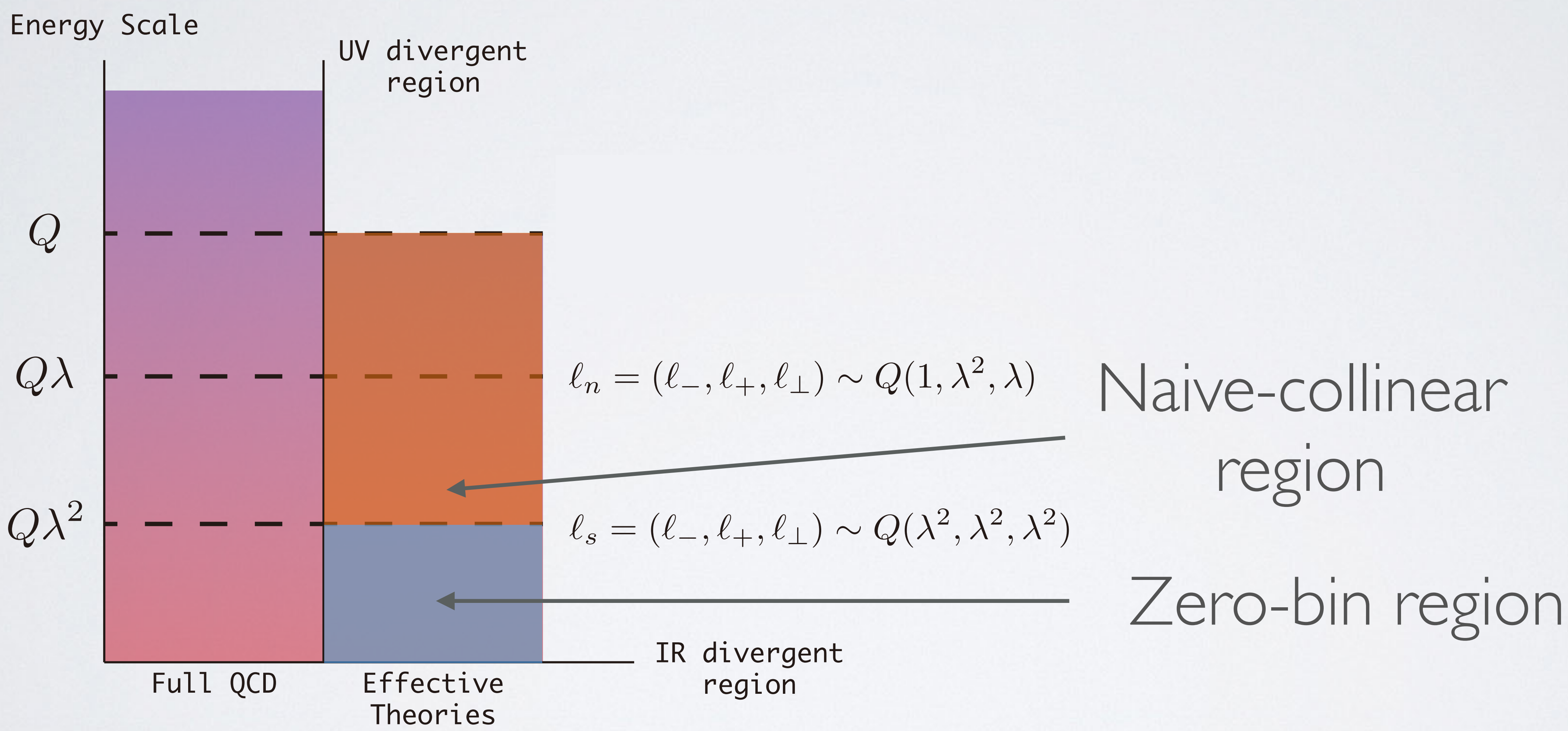
MODE COUNTING



MODE COUNTING



MODE COUNTING



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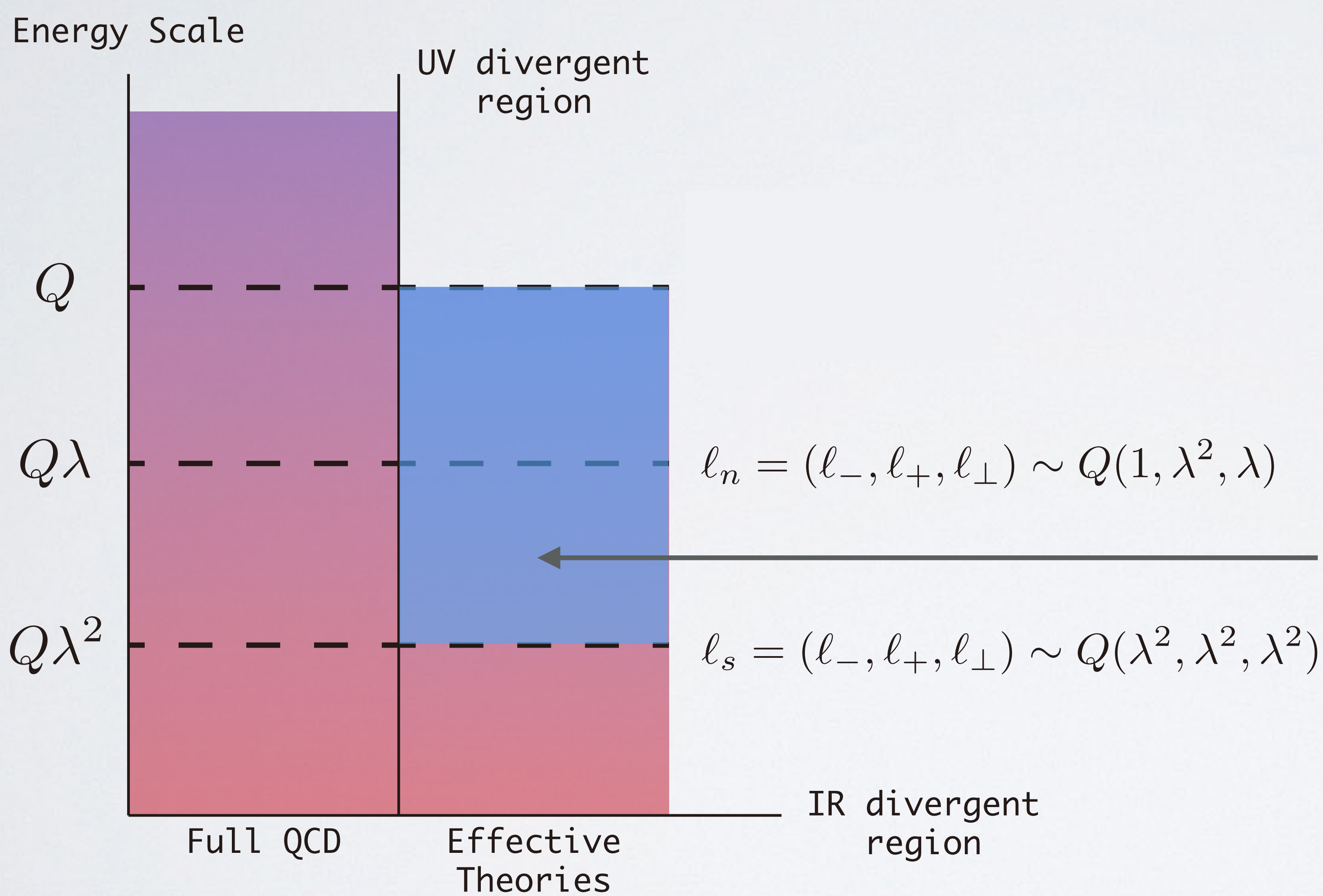
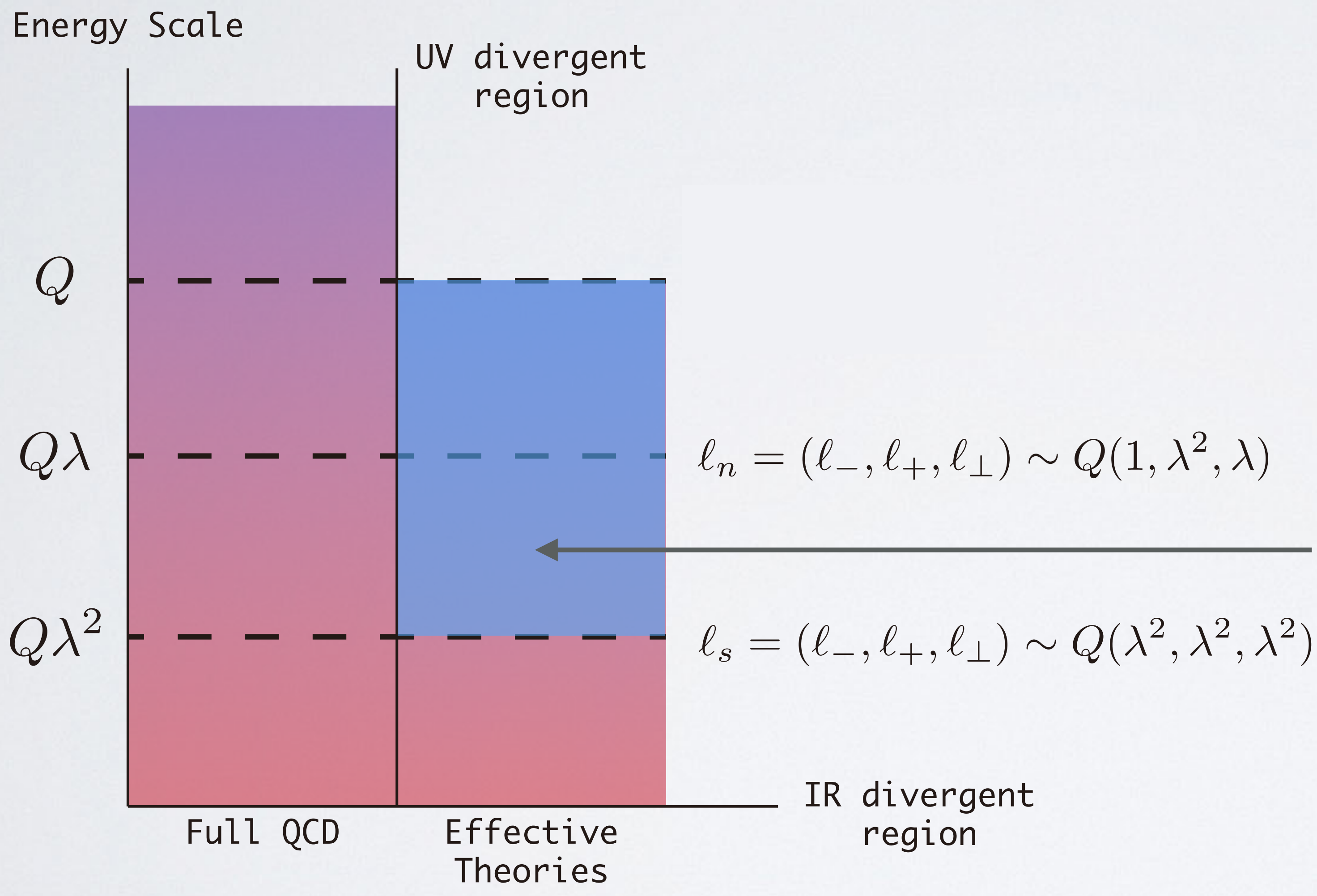


Pure-collinear region

MODE COUNTING



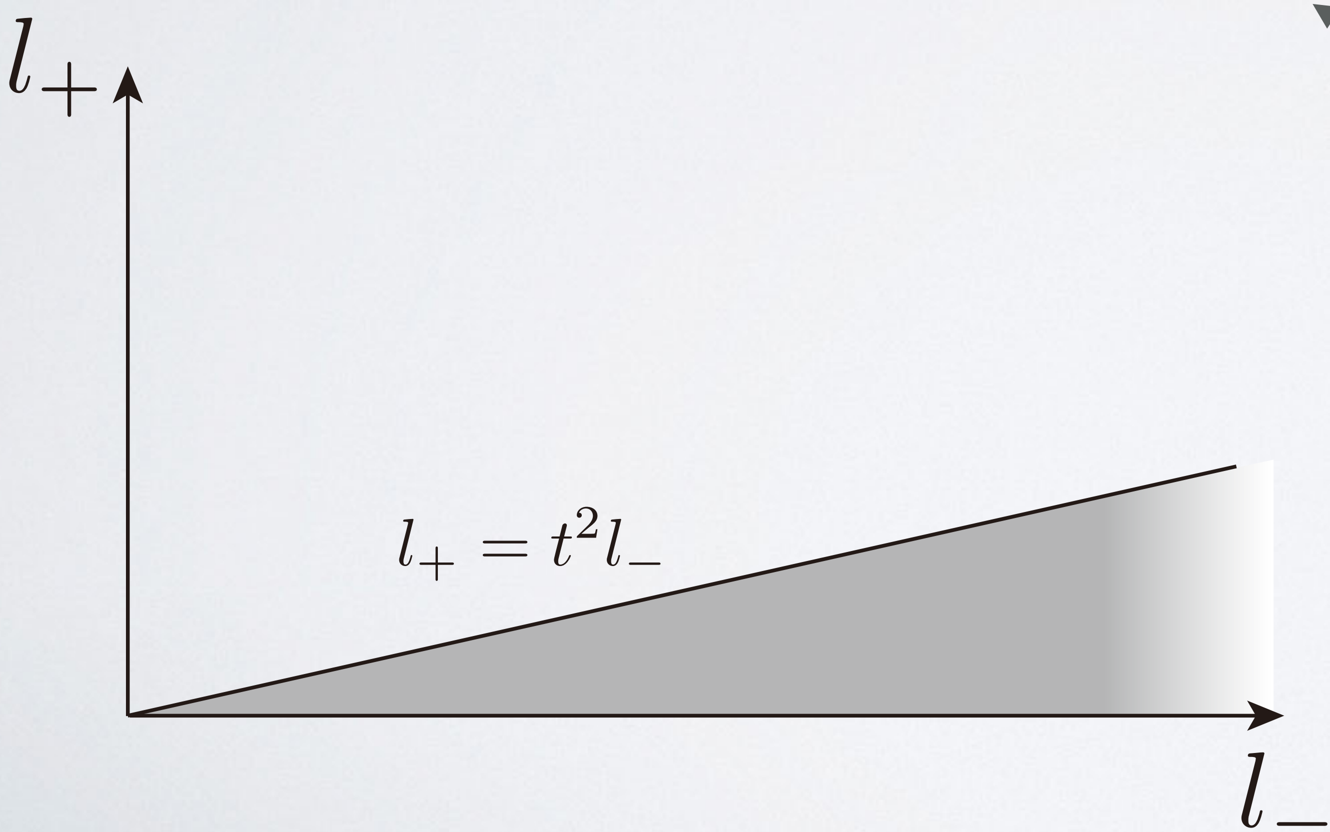
Pure-collinear region

Zero-bin subtraction!

THE JET FUNCTION -ZERO-BIN CONTRIBUTION

$$M^0_{\text{coll}} = \frac{\alpha_s C_F}{2\pi}\left[-\left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right)^2 + 2\left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right)\ln t\right]$$

Will give the correct answer?



$$l_+ \sim Q\lambda^2,\ l_- \sim Q\lambda^2$$

THE JET FUNCTION -PURE COLLINEAR CONTRIBUTION

$$\tilde{M}_{\rm coll} = \frac{\alpha_s C_F}{2\pi}\Bigg[-\frac{1}{\epsilon_{\rm IR}^2}+\frac{2}{\epsilon_{\rm IR}\epsilon_{\rm UV}}+\frac{1}{\epsilon_{\rm UV}}\left(\frac{3}{2}+\ln\frac{\mu^2}{Q^2t^2}\right)+2\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}\right)\ln t$$
$$+\frac{3}{2}\ln\frac{\mu^2}{Q^2t^2}+\frac{1}{2}\ln^2\frac{\mu^2}{Q^2t^2}+\frac{7}{2}+3\ln 2-\frac{5\pi^2}{12}\Bigg]$$

$$M^0_{\rm coll} = \frac{\alpha_s C_F}{2\pi}\left[-\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}\right)^2+2\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}\right)\ln t\right]$$

$$M_{\rm coll} = \tilde{M}_{\rm coll} - M^0_{\rm coll}$$
$$= \frac{\alpha_s C_F}{2\pi}\left[\frac{1}{\epsilon_{\rm UV}^2}+\frac{1}{\epsilon_{\rm UV}}\left(\frac{3}{2}+\ln\frac{\mu^2}{Q^2t^2}\right)+\frac{3}{2}\ln\frac{\mu^2}{Q^2t^2}+\frac{1}{2}\ln^2\frac{\mu^2}{Q^2t^2}+\frac{7}{2}+3\ln 2-\frac{5\pi^2}{12}\right]$$

IR finite result!

THE CROSS SECTION

$$H^{(1)} = \frac{\alpha_s C_F}{2\pi}\left(-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{7\pi^2}{6}\right)$$

$$S^{(1)} = \frac{\alpha_s C_F}{2\pi}\left(4\ln\frac{\mu^2}{4\beta^2 Q^2}\ln t - 4\ln^2 t - \frac{\pi^2}{3}\right)$$

$$J_n^{(1)} = J_{\bar{n}}^{(1)} = \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\frac{\mu^2}{Q^2 t^2} + \frac{1}{2}\ln^2\frac{\mu^2}{Q^2 t^2} + \frac{7}{2} + 3\ln 2 - \frac{5\pi^2}{12}\right)$$

$$\begin{aligned}\sigma^{(1)}_{e^+e^-\to 2\text{ jets}} &= J_n^{(1)} + J_{\bar{n}}^{(1)} + S^{(1)} + H^{(1)} \\ &= \sigma_0\frac{\alpha_s C_F}{\pi}\left(-4\ln 2\beta\ln t - 3\ln t - \frac{1}{2} + 3\ln 2\right)\end{aligned}$$

Consistent with the known result.

CONCLUSIONS

- To see the IR structure of QCD, one can use SCET.
- With SCET, we can systematically separate the collinear and the soft modes.
- Although the factorization approach with the cone algorithm is valid, but the factorization approach with other algorithms may not.
  - which is another topic of my work.

DIMENSIONAL REGULARIZATION

$$\int_0^\infty du(u+\delta)^{-1-\epsilon} = \frac{1}{\epsilon} - \log(\delta) + O(\epsilon), \quad (\epsilon > 0),$$

$$\int_0^\infty duu^{-1-\epsilon} = \int_0^\eta duu^{-1-\epsilon} + \int_\eta^\infty duu^{-1-\epsilon},$$

$$= -\frac{\eta^{-\epsilon}}{\epsilon}\bigg|_{\epsilon<0} + \frac{\eta^{-\epsilon}}{\epsilon}\bigg|_{\epsilon>0} = \frac{1}{\epsilon_{>0}} - \frac{1}{\epsilon_{<0}} + O(\epsilon),$$

The same UV structures

$$\equiv \frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}} + O(\epsilon),$$

$$\int_0^\infty \int_0^\infty dudv(uv)^{-1-\epsilon} = \left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)^2.$$